AP Calculus
Instructor:
2014-2015 School Year
School:
AP Exam Date: May 5, 2015

Course Description

Calculus combines an extensive study of advanced mathematical topics with the core calculus concepts of limits, derivatives, and integrals. This course will emphasize multiple representations, including graphical, numerical, analytical, and verbal representations of formulas, functions, and problems. Technology will be regularly used throughout the course and students will have access to a classroom set of TI-84 graphing calculators. By the end of the course, students will have the ability to:

- work with functions presented in multiple ways.
- understand the derivative and its uses.
- understand the definite integral and its uses.
- understand the relationship between the derivative and the definite integral.
- communicate mathematical solutions and problems in a variety of ways.
- model a real-world situation using mathematics.
- use technology to assist in solving problems.
- determine the reasonableness of solutions.
- appreciate calculus as a coherent body of knowledge.

Primary Resource

Larson, Ron and Bruce Edwards. Calculus of a Single Variable: AP* Edition, 10e ed. Boston, MA: Brooks/Cole, Cengage Learning, 2014.

Course Outline

I. Pre-Calculus Review (1 week)

- A. Graphs and Models
- B. Linear Models and Rates of Change
- C. Functions and Their Graphs
- D. Fitting Models to Data

II. Limits and Their Properties (3 weeks)

A. Finding Limits Graphically and Numerically
B. Evaluating Limits Analytically
C. Continuity and One-Sided Limits
D. Infinite Limits
E. Limits at Infinity

III. Differentiation (5 weeks)
A. The Derivative and the Tangent Line Problem
B. Basic Differentiation Rules and Rates of Change
C. Product and Quotient Rules and Higher Order Derivatives
D. The Chain Rule
E. Implicit Differentiation
F. Related Rates
G. Velocity

IV. Applications of Differentiation (3 weeks)
A. Extrema on an Interval
B. Rolle's Theorem and the Mean Value Theorem
C. Increasing and Decreasing Functions and the First Derivative Test
D. Concavity and the Second Derivative Test
E. A Summary of Curve Sketching
F. Optimization Problems
G. Newton's Method
H. Differentials

V. Integration (5 weeks)
A. Antiderivatives and Indefinite Integration
B. Area
C. Riemann Sums and Definite Integrals
D. The Fundamental Theorem of Calculus
E. Integration by Substitution
F. Numerical Integration

VI. Transcendental Functions (4 weeks)
A. The Natural Logarithmic Function: Differentiation
B. The Natural Logarithmic Function: Integration
C. Inverse Functions
D. Exponential Functions: Differentiation and Integration
E. Bases Other Than e and Applications

F. Inverse Trigonometric Functions: Differentiation
G. Inverse Trigonometric Functions: Integration
H. Hyperbolic Functions

VII. Differential Equations (2 weeks)
A. Slope Fields
B. Linearization
C. Differential Equations: Growth and Decay
D. Separation of Variables
E. First-Order Linear Differential Equations

VIII. Applications of Integration (3 weeks)
A. Area of a Region Between Two Curves
B. Volume: The Disk Method
C. Volume: The Shell Method
D. Work
E. Moments, Centers of Mass, and Centroids
F. Fluid Pressure and Fluid Force

IX. Integration Techniques (2-3 weeks)
A. Basic Integration Rules
B. Trigonometric Integrals
C. Trigonometric Substitution
D. Integration by Tables and Other Integration Techniques

X. More Integration Skills (3 weeks)
A. Euler's Method for Differential Equations
B. Logistic Differential Equation
C. Arc Length and Surfaces of Revolution
D. Integration by Parts
E. Partial Fractions
F. Indeterminate Forms and L'Hopital's Rule
G. Improper Integrals

The last unit on this outline is optional and may be skipped due to time constraints at the end of the year. Students will have 2-3 weeks for review right before the administration date of the AP exam.

This course will only be available to juniors and seniors due to our course availability. Seniors graduate in late May, so there be only two weeks of class after the exam. After the AP exam, depending on available time, students will work to finish the last unit in the outline and will complete a research project about a historical mathematician. Any juniors taking the course will complete further projects in the time remaining.

Graphing Calculators

The classroom has a full set of TI-84 Plus graphing calculators that the students will use daily in class and will be allowed to check out for use at home. Students are allowed to bring their own calculators, but few are able. Students come to class familiar with many of the functions of the calculators and knowledge of the graphing process from prior classes. I will be using the TI-SmartView for the TI-84 Plus for direct instruction on new functions of the calculators. I will not introduce the students to the nDeriv() and fnInt() functions on the calculators until second semester of the course. This will help them develop their abilities to complete problems without the calculators, and it will help them use proper notation. Some students may mistake calculator notation for proper calculus notation, and waiting to introduce the calculator functions will help to reduce that mistake.

Students will be required to use calculators for many activities and problems throughout the class. When graphing a function, students will need to determine relative maxima and minima, zeros, values, and intersections using only the calculator's capabilities. Students will also learn to use graphing calculators to explore concepts. The calculators will also be used to help students support their solutions to problems.

Teaching Strategies

There is very little direct lecture in this course. Students are encouraged to experiment with methods to solve problems and to work together throughout each class period. Students use this collaboration to make connections between the calculus concepts and knowledge they have already acquired. If these connections do not seem to be forming, I will help the class make the connections using some guiding questions. Students are encouraged (and required) to use proper vocabulary and terminology throughout the course. Vocabulary is stressed throughout lessons and used repeatedly so that students will remember new terms. Students will find themselves working with problems that are presented graphically, numerically, analytically, and verbally throughout the course and will be expected to return answers graphically, numerically, analytically, and verbally.

Students do not raise hands in this class. They must be prepared to answer a question at any given time and are encouraged to "blurt out" answers during instruction. Students also

rarely raise hands to ask questions. They are encouraged to interrupt the lesson with questions. From day one, I thank students for asking questions and remind them that they were probably not the only ones with that question.

Throughout the course, students will build a binder of notes, homework, tests, projects, and definitions. This binder will be helpful when preparing for the AP exam and when taking future math classes in college. Students are encouraged to work together on all assignments and projects and are frequently given time to work together during class. I encourage students to ask each other for help on homework questions before coming to me for help. This helps to foster their independence as learners and helps the students to understand the material in more depth.

In the weeks leading up to the AP exam, I will hold extra class meetings before school, after school, and on weekends for students to get extra help and to cover material with which they may still be struggling. These extra class sessions are not required but will be very beneficial for many students.

Student Activities

Throughout the course, students will complete many activities and projects. This will be a sampling of a few of these activities.

1. During the Pre-Calculus review unit at the beginning of the course, students will complete an activity based on arm span versus height. They will begin by collecting these measurements for everyone in the class. Depending on class size, we may gather data from a few other people as well. The students will plot these points on a calculator and experiment with various lines in order to find a line they think fits the data. They will use their line to estimate several heights and arm spans of various people. They will then analyze their results with the actual data for these people and discuss the strengths and weaknesses of their model. After this, the students will repeat the process using the linear regression capabilities of the calculators. They will then compare these results to their original results.

2. During the study of limits, students will complete an activity matching functions based on their limits. Students will receive a set of functions represented as a table (leading to a limit at a point), a graph (which shows the limit at the point), the function itself, and a sentence stating “the limit of f(x) as x approaches c is L” (given c and L depending on the function). Students will work with a partner to match the four parts of each function together using only the limits. Each function will have a different limit.

3. During the study of derivatives, students will complete a project based on optimization. They will be given no functions, only a description of a situation to be maximized or minimized. The students will need to create the function based on the information given and will then need to solve the problem. The students will be encouraged to use graphing calculators to support their solutions. There will be a variety of optimization problems available, so the whole class will not be solving the same problem. After solving the problem, students that worked on the same problem will be grouped together to compare answers and discuss differences in their answers. Finally, each group will present their problem, function, and solution to the class.

4. During the unit on applications of differentiation, students will complete a short activity on curve sketching. Each student will be given a different set of first and second derivatives of a function. Their job will be to determine the original function and all of its important characteristics, including extrema, concavity, inflection points, asymptotes, etc. The activity will be completed in three parts. For the first part, the students will receive only the graphs of the first and second derivatives. They will use their knowledge of derivatives to complete a sketch of what the original function might look like (since they do not know any initial conditions or antiderivative rules, the graphs will not be exact). For the second part, the students will receive the first and second derivatives as functions. They will use these to determine all the important characteristics of the original function and will then compare this with the graph they created in the first part. They must critique their work in part one and determine if they still agree with the graph they created. For the third part, the students will be given the function itself and will create a graph of it. The students will need to graph the function on a graphing calculator also in order to support their calculations of the important characteristics of the graph. They will then compare this function and graph with the results they found in parts one and two. They will critique their own work and determine any mistakes that were made throughout the activity. They must also explain any characteristics they may have missed about the function which were seen on the calculator.

5. When learning about the definite integral, students will complete a project. This project will require the students to create a function which lies both above and below the x-axis and to the left and right of the y-axis. The function must be at least a fourth-order polynomial. Students will determine their bounds. They will begin by graphing the function very accurately, and I will make copies of this graph for them. They will start by estimating the area between the function and the x-axis based on their bounds and the area between the function and the y-axis based on their bounds. They will be encouraged to begin by just guessing and then to get creative in their methods in order to get a more exact estimate. The second part of the project will come when we learn about Riemann sums. Students will draw rectangles and measure the areas using various sets of rectangles. The third part of

the project will require students to calculate the definite integral of their function (with respect to the x-axis). Since most functions will be extremely difficult to solve and find the definite integral with respect to the y-axis, students will need to find the areas using the trapezoidal rule and Simpson's rule. They will then be required to write a short paper explaining the various methods they used, which approximation was closest to the definite integral, and other observations they may have made throughout the project.

6. Before introducing slope fields to the students, they will complete a brief activity based on the antiderivative of a function. They will use their skills in finding antiderivatives to find the general antiderivative of a given function. The students will then graph several different functions using the general antiderivative and various values of C. They will note the similarities and differences between the graphs. They will then repeat this process, but graphing a function after being given the second derivative. They will graph using various values of C_1 and C_2. The students will then note the similarities and differences between these graphs. A discussion about why the second sets of graphs differed more than the first sets of graphs will follow this activity.

Student Evaluation

Students' progress throughout the course will be evaluated daily. Exams of evaluations include homework assignments, projects, quizzes, unit tests, a midterm exam, and several practice AP Calculus AB exams prior to the exam date. The final grade for the course will be composed of 40% homework and projects and 60% tests and quizzes. Formative assessments of students' learning will occur daily in class via warm up quizzes and/or exit quizzes.

Students will be held to high expectations. Academic integrity will be a priority and cheating will not be tolerated. In order to reduce the possibility of cheating, tests will have multiple versions, homework will not be accepted without the proper work shown, and students will be asked to verbally explain solutions on at least two quizzes. In both the midterm exam and the practice AP exams, students will be required to explain their solutions using sentences. They will be scored on these problems in a similar manner to the scoring of the AP free-response questions. Students should expect to see questions from any previously learned material on any test, quiz, or assignment. They are responsible for knowing the information throughout the school year. No extra credit will be available to students.

Other Instructional Resources

- http://www.calcchat.com/ The textbook website where students can check answers to problems.

- http://www.larsoncalculus.com/ The textbook website where students can find worked-out solutions to problems, instructional videos, and much more.
- http://www.matharticles.com/ma_calc10e.html The associated textbook website where students can read mathematical articles that give more information on topics coved in the course.
- http://dupreemath.weebly.com/ap-calculus.html Course website.
- The College Board. Released Exams for AP Calculus AB.
- Khan Academy
- Various internet resources for practice problems and worksheets
- http://education.ti.com Resource of activities related to graphing calculators.